MATH 160 FINAL EXAM SPRING 2003 **name:**

section # ______ **instructor:** ______________________

TO RECEIVE FULL CREDIT ON PROBLEMS YOU <u>MUST</u> <u>SHOW</u> <u>YOUR</u> <u>WORK</u>!!!
(If you are using a graphing calculator you must show even more work!)

1. (10) **Find an equation** of the line passing through the points (2, -1) and (0,1).

$m = \frac{-1-1}{2-0} = \frac{-2}{2} = -1$

$y + 1 = -1(x-2)$

2. (5) **Find the limit** or state that it does not exist:

$\lim_{x \to 3} \frac{x^2 - x - 6}{x - 3} = \lim_{x \to 3} \frac{(x-3)(x+2)}{x-3} = 5$

3. (10) **Find the derivative** of each of the following:

a) $y = x^5 + \frac{4}{x} + 7$

$y = x^5 + 4x^{-1} + 7$

$y' = 5x^4 - 4x^{-2}$

b) $y = \frac{1}{2}(5x+2)^{10}$

$y' = 10 \cdot \frac{1}{2}(5x+2)^9 \cdot 5$

$y' = 25(5x+2)^9$

4. (10) Let $f(x)=x^2-6x+4$. **Show** that $f'(x)=2x-6$ **using** $f'(x)=\lim_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$

$$f'(x)=\lim_{h\to 0}\frac{(x+h)^2-6(x+h)+4-(x^2-6x+4)}{h}$$

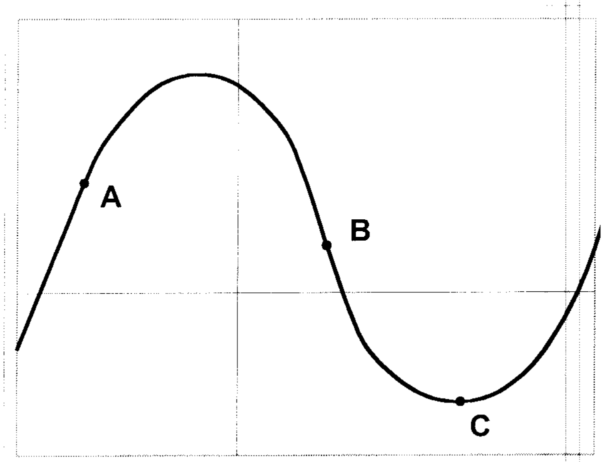
$$=\lim_{h\to 0}\frac{x^2+2xh+h^2-6x-6h+4-x^2+6x-4}{h}$$

$$=\lim_{h\to 0}\frac{2xh+h^2-6h}{h}$$

$$=\lim_{h\to 0}\frac{h[2x+h-6]}{h}$$

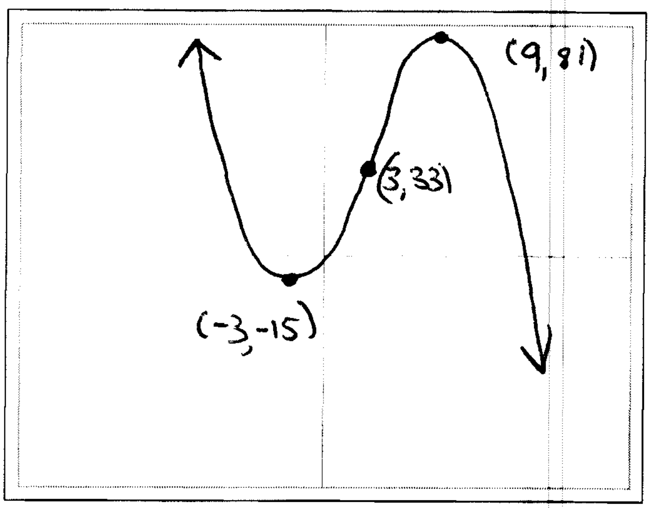
$$=\boxed{2x-6}$$

5. (10) **Fill in** each entry of the table with "**+**", "**-**", or "**0**" if $f(x)$, $f'(x)$, or $f''(x)$ is positive, negative, or zero at points A, B, and C respectively.



	A	B	C
$f(x)$	+	+	−
$f'(x)$	+	−	0
$f''(x)$	−	0	+

6. (15) Using calculus (not a graphing calculator) **find** all the relative extreme points, **determine** what kind of extrema they are, and **find** all inflection points of $f(x)=-\frac{1}{9}x^3+x^2+9x$. Using this information **graph** the function. (A correct graph without supporting work will receive no credit!)



$f'(x) = -\frac{1}{3}x^2 + 2x + 9$

$(-\frac{1}{3}x^2 + 2x + 9 = 0)(-3)$

$+x^2 - 6x - 27 = 0$

$(x-9)(x+3) = 0$

$x=9,\ x=-3$

$f''(x) = -\frac{2}{3}x + 2$

$f''(9) = -4 < 0$ so $(9, 81)$ is a max

$f''(-3) = 4 > 0$ so $(-3, -15)$ is a min

$-\frac{2}{3}x + 2 = 0 \quad 2 = \frac{2}{3}x$

$x = 3$

f'': c.up $+$ | 3 | $-$ c.dn

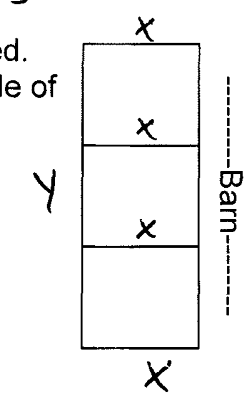
I.P. @ $(3, 33)$

List any relative maxima: $(9, 81)$

List any relative minima: $(-3, -15)$

List any inflection points: $(3, 33)$

7. (15) Three pens are to be constructed along the side of a barn as illustrated. There is 160 feet of fencing available. No fencing will be needed along the side of the barn. **Find the dimensions** that will maximize the total enclosed area.



$A = xy$

$4x + y = 160$ so $y = 160 - 4x$

$A(x) = x[160 - 4x]$

$A(x) = 160x - 4x^2$

$A'(x) = 160 - 8x = 0$

$8x = 160$

$x = 20$

$A''(x) = -8 < 0$ so $x=20$ maximizes the area

Dimensions: 20×8

8. (20) **Find the derivative** of each of the following:

a) $f(x) = x^5(x^3 + 4x - 2)$

$$f'(x) = (x^3+4x-2)(5x^4) + x^5(3x^2+4x)$$

b) $f(x) = \dfrac{6x+4}{2x-3}$

$$f'(x) = \frac{(2x-3)(6) - (6x+4)(2)}{(2x-3)^2}$$

c) $f(x) = e^{x^3-2x+1}$

$$f'(x) = e^{x^3-2x+1}(3x^2-2)$$

d) $f(x) = \ln(4x+1)$

$$f'(x) = \frac{1}{4x+1}(4)$$

9. (10) **Solve** each of the following for x:

a) $2e^{3x} = 8$

$$e^{3x} = 4$$

$$3x = \ln 4$$

$$x = \frac{\ln 4}{3}$$

b) $\ln(x-3) - 7 = 0$

$$\ln(x-3) = 7$$

$$e^7 = x-3$$

$$x = e^7+3$$

10. (10) **Find the coordinates** of each relative extreme point for $f(x)=\frac{1}{2}e^{-2x}+24x$. **Determine** if each is a relative maximum or minimum.

$f'(x) = \frac{1}{2}e^{-2x}(-2) + 24$

$f'(x) = -e^{-2x} + 24 = 0$

$e^{-2x} = 24$

$-2x = \ln 24$

$x = \frac{\ln 24}{-2}$

$y = f\left(\frac{\ln 24}{-2}\right) = \frac{1}{2}e^{-2\left(\frac{\ln 24}{2}\right)} + 24\frac{\ln 24}{-2}$

$y = \frac{1}{2}e^{\ln 24} \quad 12\ln 24$

$y = \frac{1}{2}(24) \quad 12\ln 24$

$y = 12 \quad 12\ln 24 = 12(1 \ \ln 24)$

point $(\ln 24/-2,\ 12(1 \ \ln 24))$

$f''(x) = 2e^{-2x}$ $\quad f''\left(\frac{\ln 24}{-2}\right) > 0$ concave up

so Rel MIN

11. (10) In the beginning there are 1000 cells in a bacteria culture. Let $P(t)$ be the number of cells present in the culture after t hours. Suppose that $P(t)$ satisfies the differential equation $P'(t) = 0.45P(t)$.

a) **Find the formula** for $P(t)$.

$P(t) = 1000e^{0.45t}$

b) **How many** bacteria will be present after 3 hours?

$P(3) = 1000e^{0.45(3)} \approx 386$

12. (10) A small tie shop finds that at a sales level of x ties per day its marginal profit is $MP(x) = 1.3 + 0.06x - 0.0018x^2$ dollars per tie. Also, the shop will lose \$95 per day at a sales level of $x = 0$. **Find the profit equation** for operating the shop at a sales level of x ties per day.

$P(x) = \int (1.3 + .06x - .0018x^2)\,dx$

$P(x) = 1.3x + .03x^2 - .0006x^3 + C$

$P(0) = -95$ so $C = -95$

$P(x) = 1.3x + .03x^2 - .0006x^3 - 95$

13. (10) **Evaluate** each of the following:

a) $\int_1^2 (3x^2 + 2x)dx$

$$= x^3 + x^2 \Big|_1^2 = 2^3 + 2^2 - (1^3 + 1^2)$$

$$= 8 + 4 - 2 = \boxed{10}$$

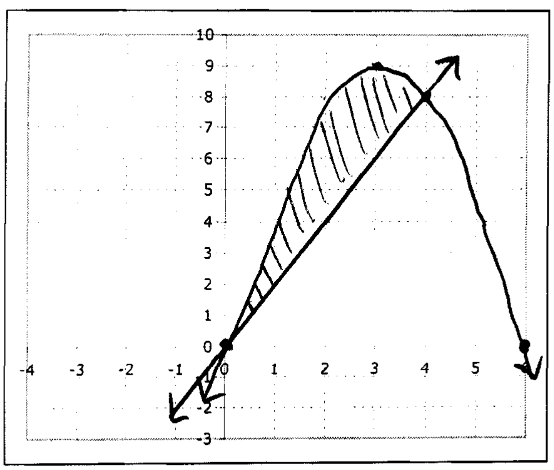
b) $\int_1^4 (3 + \sqrt{x})dx$

$$= 3x + \frac{2}{3}x^{3/2} \Big|_1^4 = 3x + \frac{2}{3}\sqrt{x}^3 \Big|_1^4$$

$$= 12 + \frac{2}{3}(8) - (3 + \frac{2}{3})$$

$$= 12 + \frac{16}{3} - 3 - \frac{2}{3} = \frac{36 + 16 - 9 - 2}{3} = \boxed{\frac{41}{3}}$$

14. (15) **Set up** the integral necessary to find the area of the region bounded by the curves $y = -x^2 + 6x$ and $y = 2x$. Also **sketch** the region. You do **not** need to evaluate the integral.



LIMITS: $-x^2 + 6x = 2x$

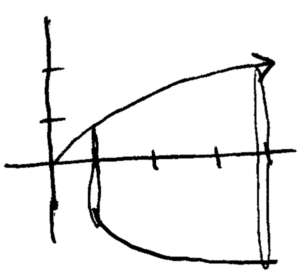
$x^2 - 4x = 0 \quad x(x-4) = 0$

$x = 0, \; x = 4$

on the interval $[0, 4]$ $-x^2 + 6x \geq 2x$

$$A = \int_0^4 [-x^2 + 6x - 2x]\,dx$$

15. (10) **Set up** the integral necessary to find the volume of the solid formed by rotating the function $f(x) = \sqrt{x}$ around the x axis from $x = 1$ to $x = 4$. You do **not** need to evaluate the integral.



$$V = \int_1^4 \pi [\sqrt{x}]^2 \, dx$$

16. (10) **Evaluate** each of the following:

a) $\int \frac{24x+1}{12x^2+x-3}dx$ $u = 12x^2 + x - 3$ $du = (24x+1)dx$

$= \int \frac{1}{u}\,du = \ln|u| + C = \boxed{\ln|12x^2+x-3| + C}$

b) $\int xe^{3x^2}dx$ $u = 3x^2$ $du = 6x\,dx$

$= \frac{1}{6}\int e^u\,du = \frac{1}{6}e^u + C = \boxed{\frac{1}{6}e^{3x^2} + C}$

17. (10) **Find all point(s)** (x,y) where $f(x,y) = -x^2 + 2y^2 + 6y - 8x + 5$ has a possible relative maximum or minimum. You do **not** need to determine whether the point(s) are maxima or minima.

$\frac{\partial f}{\partial x} = -2x - 8 = 0$

so $-2x = 8$

$\underline{x = -4}$

$\frac{\partial f}{\partial y} = 4y + 6 = 0$

so $4y = -6$

$\underline{y = -6/4 = -3/2}$

Possible relative extreme when $(x,y) = (-4, -3/2)$

18. (10) Using the method of Lagrange multipliers, **maximize** the function $f(x,y) = 3x^2 + 2xy - y^2$, subject to the constraint $5 - 2x - y = 0$.

$F(x,y,\lambda) = 3x^2 + 2xy - y^2 + \lambda(5 - 2x - y)$

$\frac{\partial F}{\partial x} = 6x + 2y - 2\lambda = 0 \rightarrow 6x + 2y = 2\lambda$ $\boxed{\lambda = 3x + y}$

$\frac{\partial F}{\partial y} = 2x - 2y - \lambda = 0 \rightarrow \boxed{\lambda = 2x - 2y}$

$3x + y = 2x - 2y$

$x = -3y$

$\frac{\partial F}{\partial \lambda} = 5 - 2x - y = 0$

$5 - 2(-3y) - y = 0$ $5 + 6y - y = 0$ $5y = -5$

$y = -1$ so $x = 3$ Maximum $= f(3,-1) = 3(9) + 2(-3) - 1 = \boxed{20}$