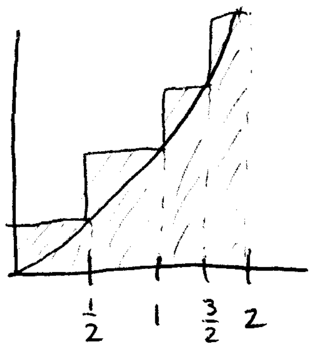
MATH 160-03
EXAM 4
NOVEMBER 30, 1999

NAME

YOU MUST SHOW YOUR WORK TO RECEIVE CREDIT

1. (10) Use the Riemann sum with $n = 4$ and the right endpoints to approximate the area under the graph of $f(x) = x^3$, $0 \le x \le 2$.



$$\Delta x = \frac{2-0}{4} = \frac{1}{2}$$

$$A \approx f(1/2)\Delta x + f(1)\Delta x + f(3/2)\Delta x + f(2)\Delta x$$

$$\approx \left(\tfrac{1}{2}\right)^3\left(\tfrac{1}{2}\right) + (1)^3\left(\tfrac{1}{2}\right) + \left(\tfrac{3}{2}\right)^3\left(\tfrac{1}{2}\right) + (2^3)\left(\tfrac{1}{2}\right)$$

$$= 25/4$$

2. (5) State the Fundamental Theorem of Calculus.

If f is continuous on the closed interval $[a, b]$ and F is an antiderivative of f then

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

3. (20) Find each of the following definite integrals:

a) $\int_0^1 6x^2 - 8x + 9\,dx$

$$6 \cdot \tfrac{1}{3}x^3 - 8 \cdot \tfrac{1}{2}x^2 + 9x \Big|_0^1 = 2x^3 - 4x^2 + 9x \Big|_0^1 = 2 - 4 + 9 - (0)$$

$$= 7$$

b) $\int_0^2 12e^{3x}\,dx$

$$= 12 \cdot \tfrac{1}{3} e^{3x} \Big|_0^2 = 4e^{3x} \Big|_0^2 = 4e^6 - 4e^0 = \underline{4e^6 - 4}$$

c) $\int_1^3 \frac{2}{x^3} + \frac{4}{x} dx$

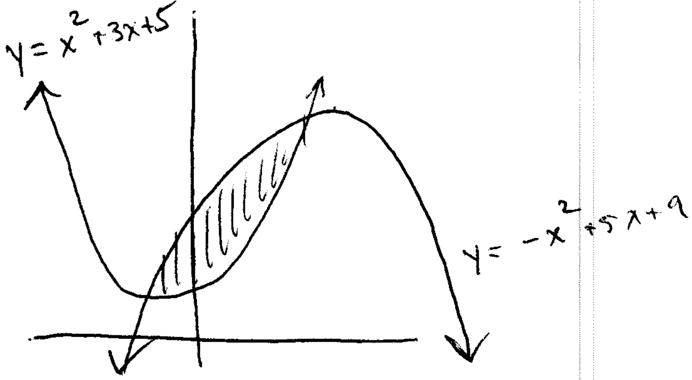
$$= \int_1^3 \left(2x^{-3} + \tfrac{4}{x}\right)dx = 2 \cdot \tfrac{1}{-2} x^{-2} + 4\ln|x| \Big|_1^3$$

$$-\frac{1}{x^2} + 4\ln|x| \Big|_1^3 = -\frac{1}{9} + 4\ln 3 - [-1 + 4\ln 1]$$

$$= -\frac{1}{9} + 4\ln 3 + 1 = \boxed{\frac{8}{9} + 4\ln 3}$$

d) $\int_1^4 6\sqrt[3]{x}\,dx$

$$= \frac{9}{2}x^{4/3}\Big|_1^4 = \frac{9}{2}\left[\sqrt[3]{x}\right]^4\Big|_1^4 = \underline{\frac{9}{2}\sqrt[3]{4}^4 - \frac{9}{2}} = \underline{18\sqrt[3]{4} - \frac{9}{2}}$$



4. (15) Find the area of the region bounded by the curves $y = x^2 + 3x + 5$ and $y = -x^2 + 5x + 9$. Include a sketch of the region.

Limits: $x^2 + 3x + 5 = -x^2 + 5x + 9$

$2x^2 - 2x - 4 = 0$

$(2x-4)(x+1) = 0$

$\boxed{x = 2 \quad \text{or} \quad x = -1}$

$$A = \int_{-1}^{2} [-x^2 + 5x + 9 - (x^2 + 3x + 5)]\,dx$$

$$= \int_{-1}^{2} (-2x^2 + 2x + 4)\,dx = -\frac{2}{3}x^3 + x^2 + 4x \Big|_{-1}^{2} = 9$$

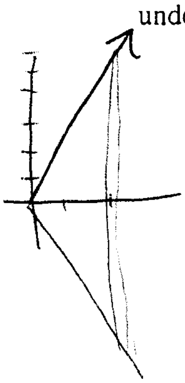
5. (10) Find the average value of the function $y = f(x) = 4x - x^2,\ 0 \le x \le 2$.

$$\frac{1}{2-0} \int_0^2 (4x - x^2)\,dx$$

$$= \frac{1}{2} \int_0^2 (4x - x^2) = \frac{1}{2}\left(2x^2 - \frac{1}{3}x^3\right)\Big|_0^2$$

$$= x^2 - \frac{1}{6}x^3 \Big|_0^2 = 2^2 - \frac{1}{6}(2)^3 = 4 - \frac{8}{6} = \frac{8}{3}$$

6.(10) Find the volume of the solid of revolution generated by the revolving about the x-axis the region under the curve $y = 3x$ from $x = 0$ to $x = 2$.



$$V = \int_0^2 \pi (3x)^2\,dx = \pi \int_0^2 9x^2\,dx$$

$$= 3\pi x^3 \Big|_0^2 = 3\pi(8) - 0 = 24\pi$$

7. (20) Find the following indefinite integrals.

a) $\int x^2\sqrt{x^3+4}\,dx$ $\quad u = x^3+4 \quad du = 3x^2\,dx$

$$= \frac{1}{3}\int \sqrt{u}\,du = \frac{1}{3}\cdot\frac{2}{3}u^{3/2} + C$$

$$= \underline{\frac{2}{9}(x^3+4)^{3/2} + C}$$

b) $\int \frac{x+4}{5x^2+40x+21}\,dx$

$u = 5x^2+40x+21 \quad du = (10x+40)dx \quad du = 10(x+4)dx$

$$= \frac{1}{10}\int \frac{1}{u}\,du = \frac{1}{10}\ln|u| + C = \underline{\frac{1}{10}\ln|5x^2+40x+21| + C}$$

c) $\int \frac{1}{(2x-6)^2}\,dx$ $\quad u = 2x-6 \quad du = 2dx$

$$= \frac{1}{2}\int u^{-2}\,du = \frac{1}{2}\cdot -1u^{-1} + C = \underline{-\frac{1}{2(2x-6)} + C}$$

d) $\int xe^{x^2+4}dx$ $\quad u = x^2+4 \quad du = 2x\,dx$

$$= \frac{1}{2}\int e^u\,du = \frac{1}{2}e^u + C = \underline{\frac{1}{2}e^{x^2+4} + C}$$

8. (10) Let $z = f(x,y) = 27 - 3x - 9y$.
EITHER a) Graph the level curves for heights -9, 0 and 9
OR b) Graph the function in 3 space.

a)