MATH 160-03 NAME

EXAM 3

NOVEMBER 2, 1999 TO RECIEVE CREDIT YOU MUST SHOW YOUR WORK

1. (10) Find the derivative of each of the following:

a) $y = (x^3+7)^2\,(x^4+9)$

$$y' = (x^4+9)\cdot 2(x^3+7)(3x^2) + (x^3+7)(4x^3)$$

b) $y = \frac{2x+3}{-4x+7}$

$$y' = \frac{(-4x+7)(2) - (2x+3)(-4)}{(-4x+7)^2}$$

2. (5) Write $(e^{3x}e^{7x})^{\frac{1}{2}}$ in the form e^{kx}.

$$e^{\frac{3}{2}x}e^{\frac{7}{2}x} = e^{\frac{10}{2}x} = e^{5x}$$

3. (10) Find the derivative of each of the following:

a) $y = e^{x^2+4x+8}$

$$y' = e^{x^2+4x+8}\cdot(2x+4)$$

b) $y = (e^{4x}+4x)^3$

$$y' = 3(e^{4x}+4x)^2\cdot(4e^{4x}+4)$$

4. (10) Find the value of x at which $f(x) = (4-3x)e^{3x}$ has a possible relative maximum or minimum point. Use the second derivative to determine which.

$$f'(x) = e^{3x}(-3) + (4-3x)\cdot 3e^{3x}$$
$$-3e^{3x} + 3e^{3x}(4-3x) = 0$$
$$3e^{3x}(-1+4-3x) = 0$$
$$3e^{3x}(3-3x) = 0 \quad \boxed{x=1}$$

$$f''(x) = 9e^{3x}(2-3x)$$
$$f''(1) = -9e^3 < 0$$

so $x=1$ gives a <u>MAX</u>

5. (5) Give the definition of $ln\, w$.

6. (10) Solve each of the following for x.

a) $3e^{7x} - 24 = 0$

$3e^{7x} = 24$

$e^{7x} = 8 \rightarrow 7x = \ln 8$

$\boxed{x = \frac{\ln 8}{7}}$

b) $2\, ln\, x = 30$.

$\ln x = 15$

$e^{15} = x$

7. (10) Find the derivative of each of the following:

a) $y = 3\, ln(x^2 - 7x + 9)$

$y' = \frac{3}{x^2 - 7x + 9} \cdot (2x - 7)$

b) $y = x^3 ln\, x$

$y' = (\ln x)(3x^2) + x^3 \cdot \frac{1}{x}$

8.(5) Simplify $e^{ln x^2 + 3\, ln y}$

$e^{\ln x^2 + \ln y^3} = e^{\ln x^2 y^3} = x^2 y^3$

9. (5) Find the derivative of $y = ln\left(\frac{(x^2-4)^3\, e^{7x}}{4x+6}\right)$.

$y = 3\ln(x^2 - 4) + \ln e^{7x} - \ln(4x + 6)$

$y = 3\ln(x^2 - 4) + 7x - \ln(4x + 6)$

$y' = \frac{3}{x^2 - 4} \cdot (2x) + 7 - \frac{1}{4x+6}(4)$

10. (10) Approximately 500 bacteria are placed in a culture. Let $P(t)$ be the number of bacteria present in the culture after t hours. Suppose that $P(t)$ satisfies the differential equation $P'(t) = .3P(t)$.

a) How many bacterial are present after 5 hours?

$$P(t) = 500e^{.3t}$$

$$P(5) = 500e^{.3(5)} \approx 2241$$

b) When will the population double?

$$1000 = 500e^{.3t}$$

$$2 = e^{.3t}$$

$$\ln 2 = .3t$$

$$t = \frac{\ln 2}{.3} \approx 2.3 \text{ hrs}$$

11. (10) Your parents bought 160 acres of farm land for $200,000 in 1969. They sell the land this year for $2,400,000. What rate of interest compounded continuously did this investment return?

Assume "this year" is 2003 so $t = 34$

$$A = Pe^{rt}$$

$$2{,}400{,}000 = 200{,}000\, e^{r(34)}$$

$$12 = e^{34r}$$

$$\ln 12 = 34r$$

$$r = \frac{\ln 12}{34} \approx .073 \text{ or } 7.3\%$$

12. (10) Find the following antiderivatives:

a) $\int 32x^3 - 24x^2 + 18x - 5\,dx$

$$= 32 \cdot \frac{1}{4}x^4 - 24 \cdot \frac{1}{3}x^3 + 18 \cdot \frac{1}{2}x^2 - 5x + C$$

$$\boxed{8x^4 - 8x^3 + 9x^2 - 5x + C}$$

b) $\int e^{3x} + x^3\,dx$

$$= \frac{1}{3}e^{3x} + \frac{1}{4}x^4 + C$$