MATH 160
OCTOBER 7,1999
EXAM 2

NAME

YOU MUST SHOW YOUR WORK TO RECEIVE CREDIT

1. (10) A ball is thrown straight up in the air from a height of 156 feet. Its height after t seconds is $s(t) = 96t - 16t^2 + 156$.

a) What is the initial velocity (when $t = 0$)?

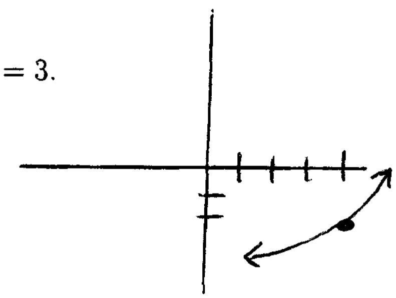
Find $s'(0)$ $\quad s'(t) = 96 - 32t$ $\quad$ so $\boxed{s'(0) = 96 \text{ ft/sec}}$

b) When will the velocity be 0?

$s'(t) = 0$

$96 - 32t = 0 \quad 96 = 32t \quad \underline{t = 3 \text{ sec}}$

2. (5) Let $y = f(x)$ be a function where $f(4) = -2$, $f'(4) = 1$, $f''(4) = 3$. Make a good sketch of $f(x)$ near $x = 4$.



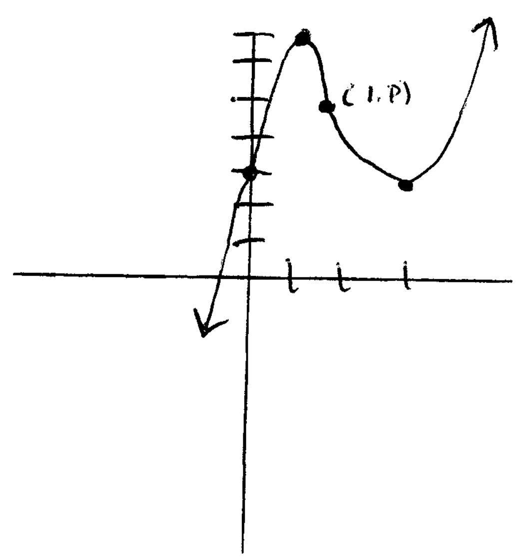
3. (15) Let $f(x) = x^3 - 6x^2 + 9x + 3$. Using calculus, determine where $f(x)$ is increasing, decreasing, and where $f(x)$ is concave up and concave down. Then sketch the graph.

$f' = 3x^2 - 12x + 9 = 0$

$3(x^2 - 4x + 3) = 0$

$3(x-3)(x-1) = 0$

$+ \quad - \quad +$ (sign line with marks at 1 and 3)



Inc on $(-\infty, 1)$ and $(3, \infty)$

Dec on $(1, 3)$

Rel max @ $x = 1$ or at $(1, 7)$

Rel min @ $x = 3$ or at $(3, 3)$

$f'' = 6x - 12 = 0$ or $x = 2$

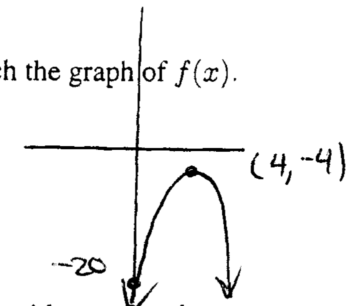
$- \quad +$ (sign line with mark at 2)

C. DN on $(-\infty, 2)$

C. UP on $(2, \infty)$

Inflection pt @ $(2, 5)$

4. (10) Let $f(x) = -x^2 + 8x - 20$. Find the one relative extreme point. Then sketch the graph of $f(x)$.



$f' = -2x + 8 = 0 \quad x = 4$

$f'' = -2 < 0$ so Rel max @ $(4, -4)$

5.(10) Let $f(x) = \frac{x}{3} + \frac{12}{x} - 1$, $x > 0$. Find the one relative extreme point for $f(x)$ with $x > 0$ and determine whether it is a relative maximum point or a relative minimum point.

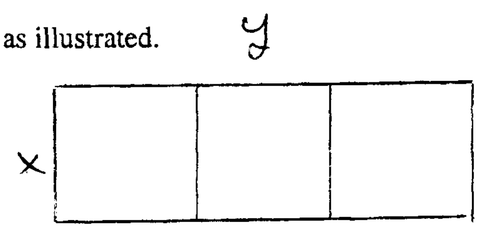
$f(x) = \frac{1}{3}x + 12x^{-1} - 1 \qquad f'(x) = \frac{1}{3} - 12x^{-2}$

$\frac{1}{3} - \frac{12}{x^2} = 0 \qquad \frac{1}{3} = \frac{12}{x^2} \qquad x^2 = 36 \qquad x = \pm 6$

$f''(x) = 24x^{-3}$ or $f''(x) = \frac{24}{x^3} \qquad f''(6) > 0$ so

MIN at $(6, 3)$

6. (15) You have 800 feet of fencing available to construct 3 pens as illustrated.
Find the dimensions that will maximize the enclosed area.
a) Identify your variables.



b) Solve the problem.

$A = xy$

$4x + 2y = 800$ so $y = 400 - 2x$

$A(x) = x(400 - 2x)$

$A(x) = 400x - 2x^2$

$A'(x) = 400 - 4x = 0 \qquad 400 = 4x \qquad x = 100$

$A'' = -4 < 0$ so A obtains a max at $x = 100$

Dimensions 100×200

7. (15) A fish farm raised 100 fingerlings in a pond. The average weight after one season was 6 pounds. For each additional fingerling placed in the pond, the average weight decreases by .05 pounds. How many fingerlings should be raised in order to maximize the total weight?

a) Identify your variables.

Let x = increase in fingerlings

b) Solve the problem.

$w(x) = (100 + x)(6 - .05x)$

$w(x) = 600 + x - .05x^2$

$w'(x) = 1 - .1x = 0 \qquad 1 = .1x \qquad$ so $x = 10$

Should raise 110 fingerlings to maximize total weight

8. (10) The cost for producing x units is $C(x) = \frac{1}{3}x^3 - 10x^2 + 120x + 200$.
Find the minimum marginal cost.

Must Find the minimum of $MC(x) = C'(x)$

$MC(x) = x^2 - 20x + 120$

$MC'(x) = 2x - 20 = 0$

$2x = 20$

$x = 10$

$MC''(x) = 2 > 0$ so $x = 10$ minimizes The Marginal cost

The minimum marginal cost is \$20/unit

9. (10) The demand function for x units of a product is $p = 400 - 10x$.
Find the value of x and the corresponding price that maximizes the revenue.

$R(x) = xp \qquad$ so $\qquad R(x) = 400x - 10x^2$

$R'(x) = 400 - 20x = 0$

$400 = 20x \qquad R'' = -20 < 0 \qquad$ so $x = 20$ is a max

$x = 20$