Math 160
May 10, 2000

Name_____
Section Number_____
Instructor_____

Do all your work on these six pages, using the backs if necessary. You must show your work, and simplify all answers, unless told otherwise. Calculators are permitted, but not required.

1. (10 pts) Find an equation of the tangent line to the curve $y = 4\sqrt{x} + 2x$ at the point $(x, y) = (4, 16)$.

2. (10 pts) For a certain commodity, the revenue from producing x units is $R(x) = 4x - \frac{1}{x}$. Find the marginal revenue at the production level $x = 30$.

3. (10 pts) Let $f(x) = x^2 + 5x - 3$. Use the limit definition of the derivative to show that $f'(x) = 2x + 5$.

4. (25 pts) Differentiate, but you need not simplify.
(a) $f(x) = x^2 - \sqrt{3}$

(b) $y = x^2 - e^{-3x}$

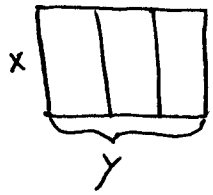
(c) $r(x) = x^2 e^x$

(d) $h(x) = \ln\left(x^2\left(x^2+1\right)^{13}\right)$

(e) $g(x) = \sqrt[3]{6x^2 - 3x}$

5. (15 pts) An open box with no top with a rectangular base whose length is twice its width is to have a volume of 36 cubic inches. Find the dimensions of the box that will minimize the amount of material used.

6. (15 pts) 6400 feet of fencing are available to build a corral that is to be divided into 3 holding pens, as shown in the drawing below. Find the dimensions of the corral that will maximize the area enclosed.



7. (10 pts) For the graph of $y = x^3 - 3x^2 - 72x + 7$, use calculus (no graphing calculators) to find the x–coordinates of the relative extreme points and determine which types of extreme points you have. Be sure to show your work!

8. (10 pts) How many years are required for an investment to double in value if it is appreciating at an annual rate of 5% compounded continuously?

9. (10 pts) Find the x–coordinate(s) where $f(x) = x^2e^{-3x}$ has a horizontal tangent.

10. (10 pts) Find the area of the region bounded by the curves $y = 3x^2 - 5x$ and $y = -5x + 3$.

11. (10 pts) Find the volume of the solid of revolution generated by revolving around the x–axis the region below $y = e^{2x}$ from $x = 1$ to $x = 3$.

12. (15 pts) Evaluate the integrals.

(a) $\int (x^3 - \frac{1}{x^2} + \sqrt{x})dx$

(b) $\int \frac{12x+3}{6x^2+3x+1}dx$

(c) $\int 3xe^{6x^2-4}dx$

13. (10 pts) Find all functions $f(x)$ with $f'(x) = 3x^2 + 5$ and $f(1) = 4$.

14. (8 pts) Find the average value of $f(x) = 2x - 5$ on $[1, 3]$.

15. (7 pts) For $f(x, y) = 8x - 4y$, draw the level curve of height 4.

16. (15 pts) Let $f(x,y) = 3y^2 - 6xy + x^3 - 9x$. Find all possible maximum and minimum points (x,y). Use the second derivative test to determine the nature of each such point. Recall $D(x,y) = \dfrac{\partial^2 f}{\partial x^2} \cdot \dfrac{\partial^2 f}{\partial y^2} - \left(\dfrac{\partial^2 f}{\partial x \partial y}\right)^2$.

17. (10 pts) Use Lagrange Multipliers to find the maximum value of $f(x,y) = 49 - x^2 - y^2$ subject to the constraint $x + 3y - 10 = 0$.