Math 160 Final Exam
May 11, 1999

Name ______________________
Section Number ________
Instructor ______________________

Do all your writing on these six pages, using the backs if necessary. You must show your work to receive credit.

1. (10 points) Find an equation of the line through $(2, -3)$ parallel to the line $3x + 5y = 6$.

2. (10 points) Find an equation of the tangent line to the graph of $f(x) = x - x^{3/2} + 1$ at $x = 9$.

3. (10 points) Let $f(x) = 3x^2 - 2x - 1$. Use the limit definition of the derivative to show that $f'(x) = 6x - 2$.

#4. (10 points) Complete each of the following differentiation formulae:

(a) $\frac{d}{dx}e^{f(x)} =$

(b) $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) =$

#5. (10 points) Solve each equation for x:

(a) $7 = 1 + 18\ln(x-2)$

(b) $3 + e^{-7x} = 25$

#6. (30 points) Find the derivative of each of the following. You need not simplify.

(a) $f(x) = 10x^8 - 8x^3 + 2\sqrt{x} - 3$

(b) $y = \frac{1}{\sqrt{3x^2+5}}$

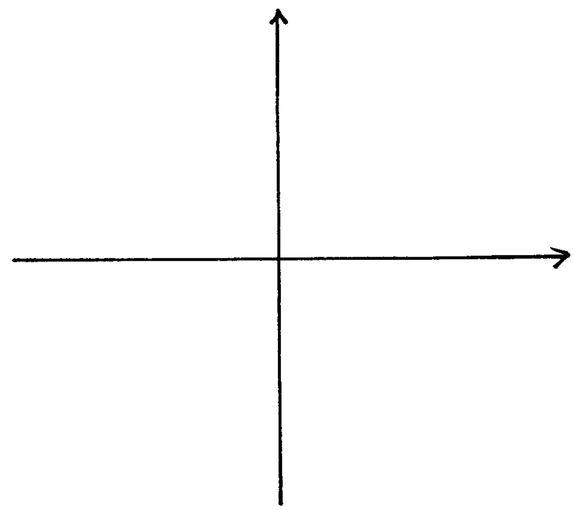
(c) $y = \frac{x^2+1}{2x^3-4x+3}$

(d) $y = x^4 \ln x$

(e) $g(x) = e^{x^2+2x}$

(f) $h(x) = \ln(7x^3 + 2x^2 + 5)$

7. (15 points) Use calculus to find all relative maximum and relative minimum points of $y = f(x) = \frac{-2}{3}x^3 + 3x^2 - 5$. Use this information to sketch the graph.



#8. (15 points) A box with a square base is to be built to have a volume of 80 cubic feet. The material for the square top and bottom of the box costs $2.50 per square foot while the material for the other four sides of the box costs $2.00 per square foot. Use calculus to find the dimensions of the box that minimize the cost of construction.

9. (10 points) The revenue for a company's commodity is $300x - 2x^2$ where x is the number of units produced. The cost of producing x units is $20 + 60x - x^2$. What number of units produced results in the maximum profit?

10. (10 points) The function $f(x) = xe^x + e^x$ has one relative minimum and no other extreme values. Find the value of x at which the minimum occurs.

11. (10 points) The world rhinocerous population decreases at a rate proportional to its size. In 1950 there were 20,000 rhinocerouses, while in 1980 there were only 5,000. How many will there be in the year 2000? Give a decimal answer.

12. (10 points) Find the area of the region between the curves $y = x^2$ and $y = x + 2$. Sketch.

13. (10 points) Find the consumers' surplus for the demand curve $p = 30 - (0.2)x^2$ at the sales level $x = 10$.

14. (15 points) Evaluate the integrals.

(a). $\int_1^2 (3x^2 + \frac{8}{x^3} + 1)\, dx$

(b). $\int \frac{x^2}{x^3 + 4}\, dx$

(c). $\int (4 + 10e^{-2x})\, dx$

15. (15 points) Let $f(x,y) = 2x^4 + 4xy + y^2$. Find all possible relative maximum and relative minimum points of $f(x,y)$. Use the second derivative test to determine the nature of each such point. If the second derivative test is inconclusive, so state.

Recall: $D(x,y) = \frac{\partial^2 f}{\partial x^2} \cdot \frac{\partial^2 f}{\partial y^2} - (\frac{\partial^2 f}{\partial x \partial y})^2$.

16. (10 points) Use Lagrange Multipliers to maximize $x^2 + xy + y^2 - 4x - 2y$ subject to the constraint $1 + x - y = 0$.