

Math 160 Final Exam
December 15, 1998

Name _____
Section Number _____
Instructor _____

Do all your writing on these six pages, using the backs if necessary. You must show your work, and simplify all answers, unless told otherwise.

1. (10 points) Find an equation of the line through the points (3,-4) and (8,6).

2. (10 points) Find an equation of the tangent line to the curve $y = x^{3/2} - x^2 + 9$ at the point (4,1).

3. (10 points) Let $f(x) = 5x^2 - 3x + 2$. Use the limit definition of the derivative to show that $f'(x) = 10x - 3$.

4. (20 points) (a). Complete the formula: $\frac{d}{dx}(f(x)g(x)) =$ (b). Find $\lim_{x \rightarrow 3} \frac{x-3}{x^2-9}$.

(c). Solve for x in $250 = 1000e^{-2.34x}$.

(d). Solve for x in $2 + 3 \ln(x+5) = 12$.

5. (30 points) Find the derivative of each of the following. You need not simplify.

(a). $y = 4x^9 + 7x^4 - 4x^3 + x^2 + 100$.

(c). $y = \frac{3x-8}{x^2+4}$.

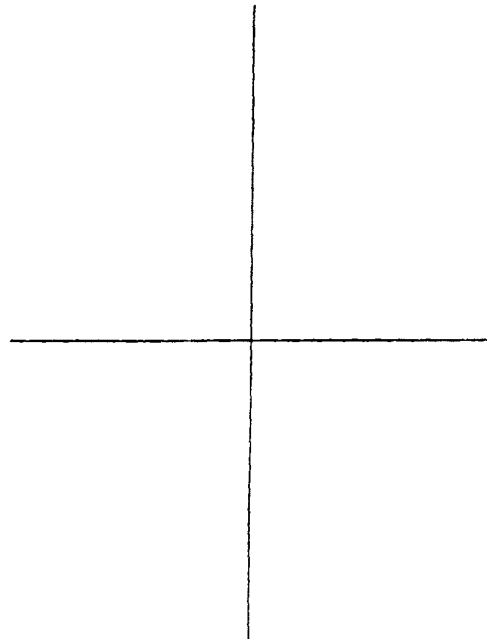
(e). $y = e^{x^3-5x}$.

(b). $y = (2x^2 + 3x - 1)^{10}$.

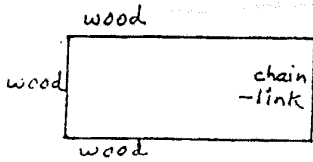
(d). $f(x) = x^2 e^x$.

(f). $g(x) = \ln(2x^3 + 1)$.

6. (15 points) Using calculus, find all relative maximum and relative minimum points and all inflection points of $y = f(x) = 2x^3 - 3x^2 - 12x + 1$. Use this information to sketch the graph.



- # 7. (15 points) A rectangular pen with area 200 square feet is to be constructed. Three sides are to be made of wood fencing and the fourth side is to be made of chain-link fencing. Wood fencing costs \$ 2 per foot and chain-link fencing costs \$ 6 per foot. Find the dimensions of the cheapest such pen.



- # 8. (10 points) The demand equation for a certain commodity is $p = 200 - 3x$, where x is the number of units and p is the price per unit in dollars. The cost function is $C(x) = 5450 + 2x$. What number of units and price per unit result in the maximum profit?
- # 9. (10 points) The function $f(x) = xe^{-2x}$ has one relative maximum and no other extreme values. Find the value of x at which the maximum occurs.
- # 10. (10 points) A population is assumed to grow at a rate proportional to its size. At a starting date the population is 5.4 million, and seven months later the population is 9.6 million. What will be the population 25 months after the starting date? Give a decimal answer.
- # 11. (10 points) Find the area of the region between the curves $y = e^x$ and $y = 4$ from $x = 0$ to $x = \ln 4$. Sketch.
- # 12. (10 points) Find the volume of the solid of revolution generated by revolving around the x -axis the following region: the area below $y = \sqrt{x+2}$ from $x = 0$ to $x = 3$.
- # 13. (15 points) Evaluate the integrals.
- (a). $\int \left(\frac{1}{x^2} + 2 \right) dx$. (b). $\int \frac{e^x}{5 + e^x} dx$. (c). $\int 4e^{-3x} dx$.
- # 14. (15 points) Let $f(x, y) = \frac{1}{5}x^5 + x^2y - \frac{1}{2}y^2$. Find all possible relative maximum and relative minimum points of $f(x, y)$. Use the second derivative test to determine the nature of each such point. If the second derivative test is inconclusive, so state.
- Recall: $D(x, y) = \frac{\partial^2 f}{\partial x^2} \cdot \frac{\partial^2 f}{\partial y^2} - \left(\frac{\partial^2 f}{\partial x \partial y} \right)^2$.
- # 15. (10 points) Use Lagrange Multipliers to find the maximum value of $f(x, y) = 2 - x^2 - y^2$ subject to the constraint $y + 2x + 1 = 0$.