

Section 1.6 Supplemental Material

Recall, in section 1.3, you used the following properties of exponents along with the Power Rule to differentiate (find derivative of power functions)

Properties:

1. $\frac{1}{x^n} = x^{-n}$

Example: If $f(x) = \frac{1}{x^{3/4}}$ then we can use property 1 to rewrite as $f(x) = x^{-3/4}$

2. $\sqrt[n]{x} = x^{1/n}$

Example: If $f(x) = \sqrt{x} - \sqrt[4]{x} + \frac{1}{\sqrt[3]{x}}$, then we can use property 2 and rewrite as $f(x) = x^{1/2} - x^{1/4} + x^{-1/3}$

The Power Rule:

Let r be any real number and let $f(x) = x^r$. Then $f'(x) = rx^{r-1}$.

Note: The derivative of a constant is 0. (If $f(x) = k$ where k is a constant, then $f'(x) = 0$.)

For each of the following functions, a) Rewrite the function as a power using property 1 and 2. b) Use the Power rule to find $f'(x)$.

1. $f(x) = \frac{3}{x}$

2. $f(x) = 2 - \sqrt{x} + \frac{5}{\sqrt[4]{x}}$

3. $f(x) = x^{5/3} - \frac{1}{x^{3/5}} + 9$

Answers: 1 a) $f(x) = 3x^{-1}$

b) $f'(x) = -3x^{-2} = \frac{-3}{x^2}$

2 a) $f(x) = 2 - x^{1/2} + 5x^{-1/4}$

b) $f'(x) = -\frac{1}{2}x^{-1/2} - \frac{5}{4}x^{-5/4} = -\frac{1}{2\sqrt{x}} - \frac{5}{4x^{5/4}}$

3 a) $f(x) = x^{5/3} - x^{-3/5} + 9$

b) $f'(x) = \frac{5}{3}x^{2/3} + \frac{3}{5}x^{-8/5} = \frac{5}{3}x^{2/3} + \frac{3}{5x^{8/5}}$

We can now extend our exponent properties to powers of functions:

Properties:

$$1. \frac{1}{f(x)^n} = f(x)^{-n}$$

Example A: If $g(x) = \frac{2}{(3x-5)^7}$ then we can rewrite g as $g(x) = 2(3x-5)^{-7}$

$$2. \sqrt[n]{f(x)} = f(x)^{1/n}$$

Example B: If $g(x) = \frac{5}{\sqrt[3]{(11x-x^7)}}$ then we can rewrite g as $g(x) = 5(11x-x^7)^{-\frac{1}{3}}$

The General Power Rule:

$$\frac{d}{dx} [g(x)^r] = r[g(x)]^{r-1} \cdot g'(x)$$

Lets take the derivative of the functions from above

$$\begin{aligned} \text{Example A: } \frac{d}{dx} \left[\frac{2}{(3x-5)^7} \right] &= \frac{d}{dx} [2(3x-5)^{-7}] \\ &= (-7)(2)(3x-5)^{-8} \cdot (3) \\ &= -42(3x-5)^{-8} = -\frac{42}{(3x-5)^8} \end{aligned}$$

First rewrite as a power

Use the General Power Rule

Simplify

$$\begin{aligned} \text{Example B: } \frac{d}{dx} \left[\frac{5}{\sqrt[3]{(11x-x^7)}} \right] \\ &= \frac{d}{dx} \left[5(11x-x^7)^{-\frac{1}{3}} \right] \\ &= \left(-\frac{1}{3}\right)(5)(11x-x^7)^{-\frac{4}{3}} \cdot (11-7x^6) \\ &= \frac{\left(-\frac{5}{3}\right)(11x-7x^6)}{(11x-x^7)^{\frac{4}{3}}} = -\frac{55x-35x^6}{3(11x-x^7)^{\frac{4}{3}}} \end{aligned}$$

First rewrite as a power

Use the General Power Rule

Simplify