

Section 1.4 Supplemental Material

In this section, it is extremely important that you can factor polynomials. If your factoring techniques are weak, see the Review of Factoring Lectures.

The topics from this section are 1) limits and 2) The limit definition of the derivative.

1) Limits:

The “interesting limits” are of the form: $\lim_{x \rightarrow a} \frac{f(x)}{g(x)}$, where $f(a) = 0$ and $g(a) = 0$. If you encounter this situation, it is a good indication that you may be able to use some algebra to simplify the limit. For example, take a look at the following limit:

$$\lim_{x \rightarrow 3} \frac{2x^2 - 5x - 3}{x^3 - 9x}$$

Note that the numerator goes to 0 as x approaches 3 and so does the denominator. This indicates that we should try some algebra to simplify the limit. Notice that we can factor both the numerator and the denominator.

$$\begin{aligned} & \lim_{x \rightarrow 3} \frac{2x^2 - 5x - 3}{x^3 - 9x} \\ &= \lim_{x \rightarrow 3} \frac{(2x+1)(x-3)}{x(x^2-9)} \quad \text{Factor} \\ &= \lim_{x \rightarrow 3} \frac{(2x+1)(x-3)}{x(x+3)(x-3)} \quad \text{Factor} \\ &= \lim_{x \rightarrow 3} \frac{(2x+1)\cancel{(x-3)}}{x(x+3)\cancel{(x-3)}} \quad \text{Cancel common factors} \\ &= \lim_{x \rightarrow 3} \frac{(2x+1)}{x(x+3)} = \frac{(2(3)+1)}{3(3+3)} = \frac{7}{18} \quad \text{Apply the rule: Limit of a Rational Function} \end{aligned}$$

2) The limit definition of the derivative.

Given a function $y = f(x)$, the derivative is defined as $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$. Remember that the notation $f(x+h)$ means that wherever we see an x in the function, we must replace the x with $x+h$.

Example: If $f(x) = x - 2x^2$, find $f'(9)$.

Solution: First we will use the limit definition of the derivative to find $f'(x)$. Once we compute $f'(x)$, we will evaluate $f'(9)$.

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{(x+h) - 2(x+h)^2 - [x - 2x^2]}{h} \\ &= \lim_{h \rightarrow 0} \frac{x+h - 2(x^2 + 2xh + h^2) - x + 2x^2}{h} \\ &= \lim_{h \rightarrow 0} \frac{x+h - 2x^2 - 4xh - 2h^2 - x + 2x^2}{h} \\ &= \lim_{h \rightarrow 0} \frac{\cancel{x} + h - \cancel{2x^2} - 4xh - 2h^2 - \cancel{x} + \cancel{2x^2}}{h} \\ &= \lim_{h \rightarrow 0} \frac{h - 4xh - 2h^2}{h} \\ &= \lim_{h \rightarrow 0} \frac{\cancel{h} (1 - 4x - 2h)}{\cancel{h}} = 1 - 4x \end{aligned}$$

Therefore, the derivative of $f(x) = x - 2x^2$ is $f'(x) = 1 - 4x$. So the derivative evaluated at $x=9$ (The slope of the line tangent to f at the point where $x=9$) is $f'(9) = 1 - 4(9) = 1 - 36 = -35$.

So, $f'(9) = -35$